

# Partons and Black Holes<sup>1</sup>

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## Abstract

A light-front renormalization group analysis is applied to study matter which falls into massive black holes, and the related problem of matter with transplankian energies. One finds that the rate of matter spreading over the black hole's horizon unexpectedly saturates the causality bound. This is related to the transverse growth behavior of transplankian particles as their longitudinal momentum increases. This growth behavior suggests a natural mechanism to implement 't Hooft's scenario that the universe is an image of data stored on a  $2 + 1$  dimensional hologram-like projection.

## 1. Introduction

In thinking about quantum gravity, and the problems of black holes, the relativity community's paradigm is basically free quantum field theory in a background geometry. But the more you spend time trying to understand what is happening near the horizon of a black hole, the less free field quantum theory seems to make sense. Instead, you find yourself forced back to the kind of thinking that we're talking about here at this workshop – partons, light-fronts, and simple qualitative pictures of what happens when

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<sup>1</sup>Based on lectures given by L.S. at the “Theory of Hadrons and Light-front QCD” workshop in Zakopane, Poland, August 1994.

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particles are at very high energies. This is a way of thinking which was popular in particle physics twenty years ago, and which has become the way of thinking for the group of people in the light-front community, a qualitative pictorial way of thinking about the constituents of matter, in situations where one is forced to think about cutoffs, and about what happens as you move the cutoffs around, uncovering more degrees of freedom of the system, fluctuating at higher frequencies and smaller distances.

Our discussion on this will be in very qualitative terms, because the technical terms describing a complete theory do not yet exist. What does exist, this way of thinking about black holes, is to some extent a collaboration (over long distances and large time scales) between L.S. and Gerard 't Hooft, and combines the ideas of 't Hooft[1], Charles Thorn[2], and L.S.[3], with the profound insights of Jacob Beckenstein concerning the maximum entropy of a region of space[4].

Other lecturers at this conference have discussed a new kind of renormalization that has to be done when doing physics in the light-front frame. This new renormalization does not have to do with large transverse momentum, or small transverse distance, but with small longitudinal momentum. Something happens for small longitudinal momentum which causes new divergences in light-front Feynman diagrams. In terms of the Feynman-Bjorken  $x$  variable, which denotes the longitudinal momentum fraction of the partons or constituents in a high energy scattering process, we have to introduce a new cutoff for very small  $x$ , and consider the effects of moving around the cutoff. The Hamiltonian  $H$  in light-front gauge is always some object which is independent of the total longitudinal momentum, divided by the total longitudinal momentum  $\eta$ . This object can be some complicated operator which typically, however, adds up to the squared mass of the system. It might involve transverse motions and ratios of longitudinal momenta, but the total  $H$  scales like  $1/\lambda$  under Lorentz boosts  $\eta \rightarrow \lambda\eta$ . We have to think about a new kind of renormalization when we Lorentz boost systems. The degrees of freedom at very low  $x$  are ultraviolet in nature, from the point of view of the light-front Hamiltonian. (While with respect to the spatial variable  $x^-$ , they are long wavelength excitations, they are very short wavelength excitations with respect to the time variable  $x^+$ .) And the tool for analyzing such a rescaling of Hamiltonians is the renormalization group. The problems of understanding matter as we increase the momentum indefinitely becomes a problem of locating all the possible fixed-points of this new renormalization group. While there are no known exact fixed-points, there are a variety of approximate behaviors of systems which will be discussed.

A very simple fixed-point was invented by Einstein and Lorentz, to describe what happens as one boosts ordinary matter. They didn't quite get it right, so along came Feynman and Bjorken, who developed a better behavior for quantum particles. There is also another fixed-point behavior which occurs in the context of free string theory. Finally, we will try to show what the correct behavior has to be in the quantum theory of gravity, why it has to be the correct behavior, and what it has to do with black holes, and the

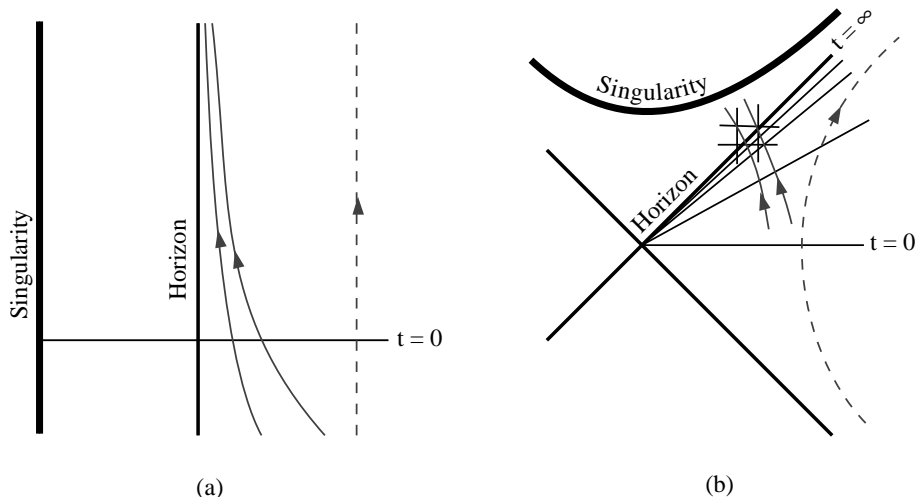


Figure 1: (a) Black hole in radial coordinates. (b) in Kruskal coordinates. Two of the incoming particles in the diagrams are falling into the black hole.

paradoxes of black holes. We will also briefly explain how this leads one to believe that the universe is not what one normally thinks of as three dimensional, but is, in a way, two dimensional. In this sense, the universe is an illusion!

## 2. Black Holes and Infalling Matter

Before discussing the fixed-point behaviors, let us consider what all this has to do with black holes. Consider figure 1(a), which is a picture of a massive black hole in radial coordinates. On the left of the diagram is the singularity, where bad things happen to you, and further towards the center of the diagram is the horizon. As is well known, if you are inside the horizon, all light rays and all time-like trajectories inevitably lead into the singularity. The horizon separates that region of space from the region where you at least have a chance of getting out.

Any matter falling into the black hole, as seen by an observer outside the black hole in his coordinate frame and using the Schwarzschild (asymptotic observer) time, will never be seen to cross the horizon. (The trajectories of two such observed particles is shown in figure 1(a).) As the matter falls in, in the frame of reference of the static observer, the momentum increases. It increases for the same reason that the momentum of any object increases when it falls down, and it's velocity increases, i.e. it gets boosted. It's momentum in fact increases very rapidly with time – exponentially rapidly. Now consider the Kruskal

diagram (figure 1(b)) of exactly the same situation. Notice that the singularity is space-like and not time-like in this diagram<sup>4</sup>. On this diagram light rays move on 45 degree lines, and if you are in the upper left of the 45 degree Horizon line, you are trapped, and you have no choice but to fall into the singularity. If you are in the outer wedge to the right of the Horizon line, you have a chance of escaping to infinity. There is a one-to-one correspondence between the two diagrams. On the Kruskal diagram, lines of constant time are lines of constant angle. The relationship between one time and another time is a boost, so time is simply boost angle in this representation. They get denser and denser as they approach the horizon line. The horizon is  $t = \infty$ , where  $t$  is the time as seen by a stationary observer outside the black hole. A fixed static observer moves on the dashed hyperbola of fig. 1(b), and she cannot look in or ever see anything behind the horizon. Looking back along his light-fronts, all she can ever see are particles approaching the horizon, but never passing through it, because by the time they get to the horizon, she's at her  $t = \infty$ . A second coordinate patch of figure 1(b) is regular from the point of view of infalling matter. In terms of ordinary cartesian coordinates its metric is given by

$$ds^2 = dT^2 - dZ^2 - dX^i dX^i, \quad (1)$$

while in terms of Schwarzschild coordinates the same space-time is given by

$$ds^2 = \left(\frac{dt}{4MG}\right)^2 \rho^2 - d\rho^2 - dX^i dX^i, \quad (2)$$

where we have used the coordinate  $\rho$  which denotes proper distance from the horizon instead of the usual radial coordinate  $r$ . The Minkowski and Schwarzschild coordinates are related by

$$Z = \rho \cosh(t/4MG), \quad (3)$$

$$T = \rho \sinh(t/4MG). \quad (4)$$

There is a big mismatch between the two coordinate systems as time goes on. The mismatch is a huge boost. That is, the effect of a Schwarzschild time translation on the Minkowski coordinates is a boost

$$\Delta\omega = \frac{\Delta t}{4MG}, \quad (5)$$

and as time goes on, the outside observers coordinate system and the coordinate system associated with the infalling matter become infinitely Lorentz boosted relative to each other. The horizon of the black hole is the surface  $t = \infty$ . This point can be made in

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<sup>4</sup>That has led some people to ask not where, but when is the singularity. The singularity is in a sense at a place in time.

another way. As the particle falls toward the horizon, it is accelerated from the point of view of a static observer. The momentum of the particle increases like

$$P \rightarrow e^{t/4MG} . \quad (6)$$

To give a proper account of the particle from the Schwarzschild frame we must have a description which is valid at ever increasing momentum.

Black holes, as is well known, evaporate. They radiate photons, and other things, and at the same time, matter may be falling in. One may imagine throwing in matter at the same rate that the black hole evaporates in order to think of a fixed size black hole. One keeps throwing in more and more matter – graduate students, cities, planets, generally huge amounts of matter, as long as we do it slowly enough so that the black hole won't increase its size. Enormous amounts of matter accumulate at the horizon. How does the horizon manage to hold all that material? How does it manage to squeeze that much in? The answer, according to the usual picture, is that the matter Lorentz contracts. Because the momentum given by eqn. (6) becomes so large so quickly as the objects fall in, they "pancake" and get flattened. They get so incredibly flattened, that you can apparently put any amount of matter on the horizon. The Hawking radiation apparently has nothing to do with the incoming matter – it is produced by curvature at the horizon, and it not the matter that is falling in, boiling, and coming back out. While this decoupling between infalling matter and Hawking radiation is almost certainly wrong, the point of this discussion is to make one realize that in the study of matter falling onto the horizon, one is encountering huge boosts. How long do we have to follow this matter? For the purposes of this discussion, we have to follow it for a time comparable to the lifetime of the black hole. The lifetime of a black hole is proportional to  $M^3$ , so if we wait long enough, we will find that the momentum of the infalling matter has grown to

$$P_{\text{lifetime}} = e^{M^2 G} . \quad (7)$$

We have to track the matter until it's momentum is an exponential of the black hole's mass squared in Planck units. A huge amount of momentum is involved even if the mass of the black hole is a hundred or so Planck masses. This energy is vastly bigger than the entire energy in the universe! Therefore, in order to understand and study the properties matter falling into a black hole, we have to understand the boost properties of matter far beyond the Planck scale.

### 3. Boost Properties of Matter

The boost properties of matter are described by the fixed points of a new kind of renormalization group. Let us consider some of the qualitative aspects of the speculations

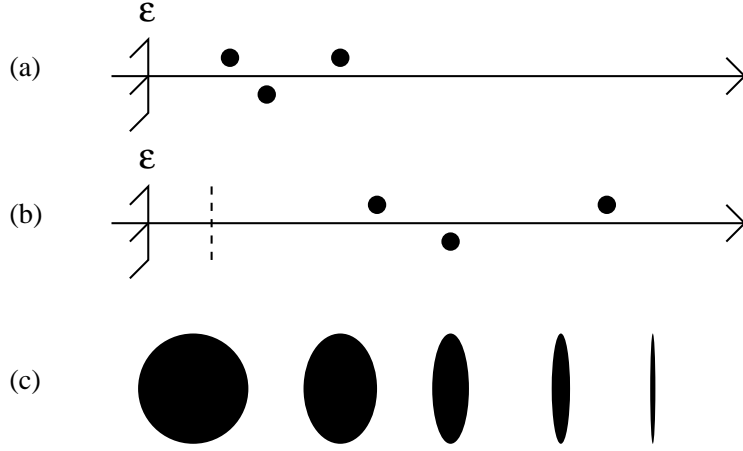


Figure 2: The Einstein - Lorentz fixed point under boosts. (a) Initial Parton distribution on the longitudinal momentum axis,  $\epsilon$  is a momentum cutoff. (b) Parton distribution after a boost which doubles constituent momentum. (c) Pictures of the matter undergoing greater and greater boosts. A pancake picture of the matter emerges.

that have already been put forward about these fixed points<sup>5</sup> Consider the longitudinal momentum of a particle. We need to cut off very small longitudinal momentum with a cutoff  $\epsilon$ , because that region corresponds to very ultraviolet behavior in the light-front frame[5]. Physically, we imagine that we have a set of detectors or apparatuses that are sensitive to frequencies up to some maximum of order  $M^2\epsilon^{-1}$ . An appropriate description should be possible with longitudinal momenta cutoff at  $p^+ = \epsilon$ . It is important to keep in mind this connection between the cutoff procedure and an apparatus.

#### A. The Einstein Lorentz Fixed Point

In the picture of matter developed by Einstein and Lorentz, the particles consist of only a few ‘valence’ partons, shown on the longitudinal momentum axis in fig. 2(a). When we boost the matter by some factor, each parton constituent of the matter gets boosted by the same factor. In this E-L picture, transverse position of the partons doesn’t change. So when the momentum of the matter is doubled, the longitudinal momentum of the partons which make of the matter is doubled. One has also doubled the differences between the momentum of the partons, and doubled the fluctuation in momentum. So by the uncertainty principle, you have contracted everything by the same factor of two. The

<sup>5</sup>They were not called fixed points when they were originally worked out, but now it is worthwhile to think of them this way.

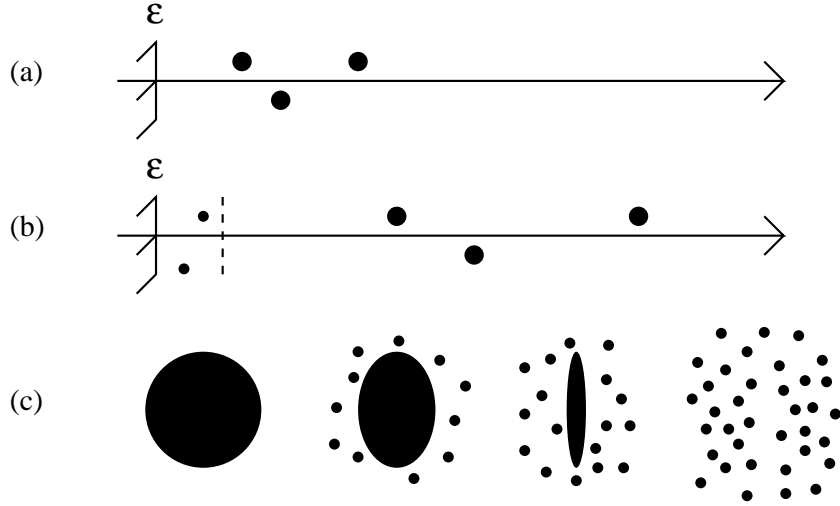


Figure 3: The Feynman - Bjorken fixed point under boosts. (a) Initial valence parton distribution on the longitudinal momentum axis. (b) Parton distribution after a boost which doubles constituent momentum. Wee partons appear from the vacuum effects below the old (now boosted) cutoff and above  $\epsilon$ . (c) Pictures of the matter undergoing greater and greater boosts. A pancake picture of the matter emerges, with an unboosted and transversally widening cloud of wee partons.

E-L fixed point is described by the behavior of matter shown in figure 2(b) for different values of the boosts. When the particle is at rest, it looks like a sphere. When the particle is moving, it looks like an ellipsoid. As it moves faster, it looks like an even flatter ellipsoid. Send it to extremely high energy, and it looks like a flat pancake, with infinitely thin dimensions. That is the E-L fixed point. If the cutoff  $\epsilon$  is sufficiently small the probability to find a parton with  $p^+ < \epsilon$  is vanishingly small. Boosting the system is trivial, as each parton shifts to its boosted position. Transverse sizes are invariant and longitudinal sizes contract. Most likely no real 3+1 dimensional quantum field theory works this way.

## B. The Feynman Bjorken Fixed Point

The next fixed point is the Feynman-Bjorken (F-B) fixed point[6,7]. It is a lot like the E-L fixed point with respect to the valence partons. These partons carry the information of the matter that tells it that it's a proton and not a neutron – isospin, charge, etc, and there is only a few of them. When you double the momentum of the matter, these valence partons also double their momentum. If we drew the same sequence of boosts

for the valence partons, we would expect the same E-L pancake behavior. But according to Feynman and Bjorken, maybe there are degrees of freedom behind the  $\epsilon$  momentum cutoff, which we don't care about because they are fluctuating too rapidly for us to see. Now we boost everybody, and some degrees of freedom which might have been behind this cutoff now come out into direct view<sup>6</sup>, where our physics should be able to see it. The very high frequency fluctuations become visible when we further boost the system because they slow down. In this model of hardons, the the number of partons per unit  $p^+$  behaves like

$$\frac{dn}{dp^+} \sim \frac{1}{p^+} \quad (8)$$

So, in the F-B fixed point picture, every time we boost matter, we pull some more partons out of the vacuum. In other words the boost operator must contain an extra term which acts as a source of partons of low  $p^+$ . These are the partons that Feynman called the 'wee' partons. The wee partons create interesting anomalies in the behavior of matter under boosts. For example, because they always carry low longitudinal momentum they contribute a cloud which does not Lorentz contract. Also, while Feynman and Bjorken did not say this, one believes that they tend to spread out a little in the transverse direction, as the system is boosted. For example, in simple Regge approximations their average distance from the center of mass satisfies[8]

$$R_{\perp}^2(\text{wee}) \sim \log \frac{p^+(\text{tot})}{\epsilon} \quad (9)$$

So this improved F-B fixed point is the following. At rest, the matter is a sphere with no wee partons. Boost it and the valence stuff behaves as in the E-L picture and pancakes, but the wee partons form a cloud around it which does not Lorentz pancake, but may spread out a little bit transversely. This is shown in figure 3.

### C. The Kogut-Susskind-Lipatov-Alterelli-Parisi Fixed Point

The next fixed point to consider is the Kogut-Susskind-Lipatov-Alterelli-Parisi [9] (KSLAP) one, which attempts to treat another aspect of this problem. If you look at things with better and better transverse resolution, you begin to see matter consisting of smaller and smaller parts. At low momentum, a meson would look like two partons, but if you increase the transverse momentum, you might see "new" wee partons, but also if you look at the original valence partons more carefully, you might also see that they consist of partons within partons. For example, a quark might spend part of its time being a quark and a gluon. A gluon spends part of its time being a pair of quarks. This is shown schematically in figure 4. As we boost the system so that the parton has a much larger

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<sup>6</sup>The cutoff is kept in the same place under boosts. Note that we could also keep the momentum fixed and decrease the cutoff.



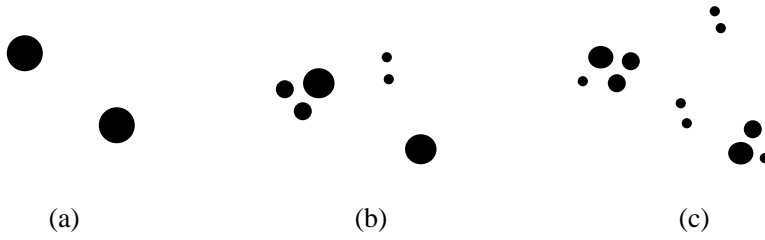


Figure 4: The behavior of matter with respect to boosts in the KSLAP fixed point picture. As the resolution increases (fig. (a)  $\rightarrow$  fig. (c)), you see both new wee partons, and valence partons that spend time as multi-parton configurations.

$p^+$ , the transverse phase space for it to split is much bigger. If the theory has transverse divergences then that probability will become large as the system is boosted. Eventually the parton will be replaced by two or more partons closely spaced in transverse space. The effect continues as the system is boosted so that the partons reveal transverse fine structure within structure ad infinitum. One can do a great deal of physics in the study of deep inelastic scale violations by thinking naively in this physical way. In fact, one important consequence for high energy scattering is the existence of processes involving large transverse momentum jets.

#### D. The String Theory Fixed Point

Let's think of strings as being made up of balls and springs. (Forget for the moment any fancy mathematics.) What happens as you add more balls and springs, in an effort to try to define a string? You add more and more normal modes of higher and higher frequency. To understand the low frequency aspects of the string, you can simply think of it as a couple of balls and springs. (From now on, low momentum means Planck scale!) If you want to get all the high frequency modes right, you have to add a lot of balls and springs. At some relatively low Planckian momentum, let's assume a particle consists of two partons – two balls and a spring. This is just our longitudinal momentum cutoff because we don't see very high frequencies. As we probe the string at higher and higher momenta, we need to add more and more balls and springs, and it seems to the observer that the original constituents are breaking up into more constituents, simply of the same low momentum, with the number of total constituents being the total momentum of the particle. The normal modes of the string fluctuate in the transverse direction, yet you see more of them as you probe higher and higher light-front energy. In string theory, transverse momentum never get big, and we are also seeing here that it also says soft longitudinally. So, instead of boosting the particle by increasing the momentum of the

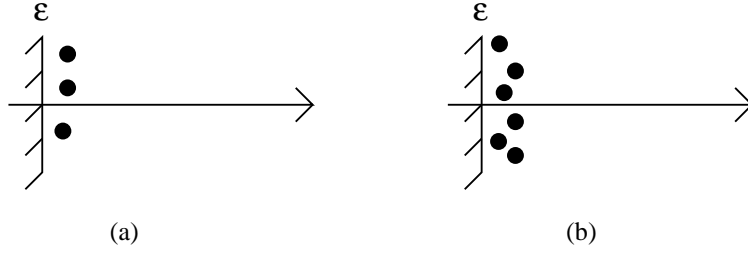


Figure 5: The string theory fixed-point. As the momentum increases (fig. (a)  $\rightarrow$  fig. (b)), more partons appear. All partons are wee, and their number is proportional to the transverse radius squared. The momentum of the string grows like  $P = 2^N$ , where  $N$  is the number of partons.

constituents, you just increase the number of constituents. Notice that since you never make large momentum constituents, there is no reason to think that this object Lorentz contracts. Every parton is wee inside the string. Charles Thorn calls this just the fact that the string is made of wee partons[2]. Mathematically, this is related to ‘conformal invariance’ of the string. The pattern of growth in transverse position is shown in figure 5. It’s called “branching diffusion”. Each time you double the momentum, each constituent bifurcates. It’s like the evolution of a petri dish full of single organisms, except in free string theory those single organisms don’t push each other out of the way. A low energy string might consist of just two balls, with a size and separation characteristic of the string scale. Double the momentum and each ball bifurcates into two balls with some random orientation in the transverse direction, each one of which is about the same size as the original one, and the distance between them is about the same characteristic string scale. Eventually, for a large amount of bifurcations, the total momentum is

$$P^+(\text{tot}) = \epsilon 2^n , \quad (10)$$

where  $n$  is the number of partons. The mean square radius of this object is defined as follows: each time it bifurcates, focus onto one of the bifurcation products randomly, and go to the next one, etc. As you do so, you will be forming a random walk in the transverse plane in coordinate space. So the mean square size of the object will grow like

$$R^2 = \ell_s^2 n = \ell_s^2 \log(P^+/\epsilon) , \quad (11)$$

where  $\ell_s$  is some characteristic string length scale that is of order of, or greater than, the Planck length<sup>7</sup>. The precise relation is

$$\ell_{\text{Planck}} = g \ell_s \quad (12)$$

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<sup>7</sup> Note that eqn. (11) is a Reggie pole formula, and explains why string theory has Reggie pole behavior.

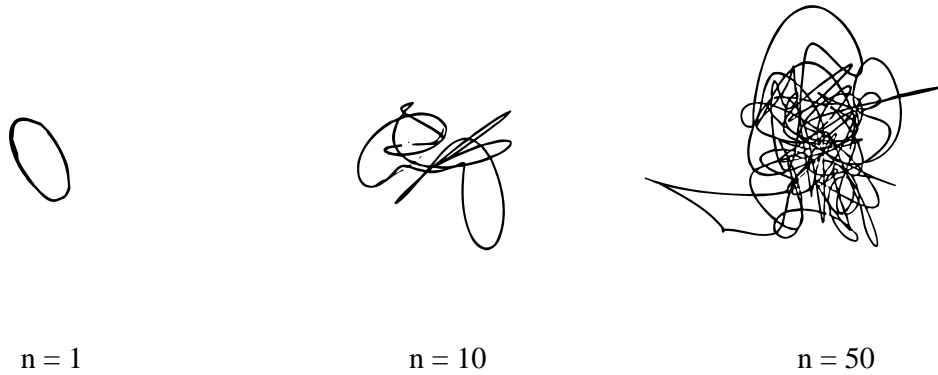


Figure 6: Snapshots of a string's transverse extent as more and more modes  $n$  are included. Ultimately it becomes space filling as  $n$  goes to infinity.

where the string coupling  $g$  is much less than one for weakly coupled strings. We will basically assume that strings are weakly coupled in this lecture.

Let's see that the mean square radius of an object does indeed increase in this way in string theory, and how it's connected to this new kind of renormalization group thinking. A string in the transverse plane is parameterized as

$$X^\perp(\sigma) = X_{\text{cm}}^\perp + \sum_l \frac{a^\dagger(l)}{\sqrt{l}} e^{il(\sigma+\tau)} + \dots \quad (13)$$

where  $0 < \sigma < 2\pi$  labels the position along the closed string, for fixed light-front time  $\tau$ , and  $X_{\text{cm}}^\perp$  labels the center of mass of the string. The rest of the expansion in eqn. (xperp) is a bunch of normal mode oscillations. Each of these oscillations has higher and higher frequencies in light-front time. The mean squared radius of the probability distribution of a point on the string, where we have subtracted the center of mass contribution, is given by the formula

$$R^2 = \sum_l \frac{\langle a^\dagger(l)a(l) + a(l)a^\dagger \rangle}{l} \sim \sum_l 1/l \rightarrow \log \infty . \quad (14)$$

Therefore, it is a complete lie that strings are little objects about as big as the Planck scale <sup>8</sup>. However, what is wrong with this picture is that the infinity is coming from the incredibly high modes of oscillation that we can't see. They are behind the low  $k^+$  cutoff in this sense. How many modes should we account for? The number of modes we should account for is comparable to the momentum, because the time dilation factor of a fast object is comparable to its momentum. Therefore we find that

$$R^2 \rightarrow \log P , \quad (15)$$

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<sup>8</sup> This is what made string theory fail as a theory of hadrons, since hadrons are not infinitely big.

and that is the same rate of growth given by the bifurcation for the string fixed point. The total length of string grows as  $P$  itself, so the momentum per unit length is approximately constant. Figure 6 shows some snapshots of a string as you include higher and higher normal modes[10]. These are Monte Carlo samplings of string configurations drawn from the wave functions of strings with given numbers of included modes. We can think of these pictures in several different ways. We can think of them as strings where you include more and more modes. Or, we can think of this as strings at even increasing momentum. Or, we can think of these as the description of a string as falls towards a black hole.

#### 4. The String Picture of High Energy Scattering and Matter Falling into Black Holes

Now let us discuss high energy scattering amplitudes, and let's see why string theory is quantum gravity and not quantum 'something else'. Take two particles scattering off each other, at first in a weakly coupled approximation, and we won't worry about unitarity for the moment. One particle has momentum  $P$  and the other momentum  $Q$  in string units. Following the string theory picture of the constituents, the first contains  $P$  partons, and the second  $Q$  partons. It turns out to be easier to think about the interaction in the center of mass frame of the two particles, although we still have the picture of their constituents in the infinite momentum frame. We assume that the scattering amplitude is the sum of parton – parton scattering amplitudes, and will think of this later on as gravitational scattering, but always gravitational scattering between low momentum constituents. The energy of the constituents is not increasing, and the scattering amplitude of the constituents is independent of the momentum  $P$  and  $Q$ . We therefore expect the forward coherent scattering amplitude will go like  $S = P * Q$ , where  $S$  is the Mandelstam  $S$  variable. The energy of the system is roughly  $E^2 = P * Q$ , so  $S$  goes like  $E^2$ . In Reggie theory, a pole with intercept corresponding to spin  $s$  exchange contributes as  $S = E^s$ , so our result is exactly the rate of growth of the scattering amplitude for a spin two exchange. That is the graviton trajectory, so if this theory can be made to fly, it will automatically have a graviton Reggie trajectory.

Now this is far from unitary as it stands, since the particle grows only logarithmically in transverse size as you increase the number of partons, but we are saying that the forward cross section (proportional to  $S$  by the optical theorem) grows linearly as you increase the number of partons. In a unitary theory, a cross section cannot be bigger than the particles which are scattering, they can't be bigger than the geometric size of the object. We will resolve this paradox in section 5.

In the standard picture of the relativists, who think about free particle quantum field theory, the size of a particle is longitudinally boost invariant, so as it falls toward a black hole, a point particle remains a point-like object which asymptotically approaches the horizon. What does string theory say? As you approach the horizon,  $R^2 = \ell_s^2 \log P$ , from

eqn. (11). We also know from eqn. (6) that as a particle falls towards the horizon, its momentum increases exponentially rapidly, so the radius increases rather substantially as

$$R^2 = \frac{\ell_s^2 t}{4MG} . \quad (16)$$

(The  $1/4MG$  factor is just the time dilation factor as seen from spatial infinity. An observer near the horizon would not see this factor.) This growth is characteristic of diffusion. Now recall that the radius of a black hole is  $R_{\text{schw}} = 2MG$ , so we can calculate how long it will take for the string to spread across the entire black hole. We find that

$$t_{\text{spreading}} = \frac{G^3 M^3}{\ell_s^2} . \quad (17)$$

This time to spread across the entire black hole is small compared to  $G^3 M^3 / \ell_{\text{Planck}}^2$ , which in turn is the time it takes to evaporate the black hole altogether. With this very modest log growth in the string size, there is enough time for the string to spread across the entire horizon.

This is a particularly strange result for the following reason. Somebody falling with the string does not get to see these increasing number of normal modes<sup>9</sup> So the matter which falls in doesn't know that it's spreading, or "melting", but an outside observer can see this quite clearly. As long as the observer falling into the horizon cannot send out a message to the asymptotic observer, there will be no paradox as to the description of its extent along the horizon.

## 5. Unitarity and Beckenstein

Consider two particles, which are balls of wound up string, whose radius grows only logarithmically in energy as shown in the previous section. We seem to have calculated that their elastic cross section grows as a power law in the energy, which is a violation of unitarity. According to Froissart[11], the cross section of such an event should never grow faster than a logarithm of the energy squared<sup>10</sup>, so there must be huge shadowing corrections if in the lowest order, we seem to find the power law. At least in ordinary

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<sup>9</sup> Consider throwing a hummingbird into a black hole. As it approaches the black hole, the motion of its wings slow down, because it is being time dilated, or alternatively, because it's increasing its momentum. So we see it going from just a little body with nothing else there, to a thing with some wings. What does a hummingbird think about all this? Not much, because it doesn't know it has wings since it has never been time dilated relative to itself, and it can't see its wings as it falls into the horizon either.

<sup>10</sup> The Froissart bound is  $\sigma(\text{total}) < C \log^2 E$ , as the incident particle energy  $E \rightarrow \infty$ . The rigorous derivation of this bound is based on unitarity and the domain of convergence of the partial wave series for the imaginary part of the amplitude. If the partial wave amplitudes are not normalizable due to long range interactions, then the derivation can be invalidated.

physics that would be the case as we try to stack up more and more matter onto the incident bundles of matter. The other alternative is that the matter, because of strong interactions, pushes itself away, so that the geometrical radius grows faster than a log. The rate of growth that was described in section 3D is what happens when you have a bunch of free micro-organisms growing on a petri dish, which don't push each other out of the way. At some point, you get a huge density of them in the center, because the radius is only growing logarithmically, but the number is growing linearly. They are going to get incredibly dense. When they get this dense, no matter how small the coupling constant, they will start to interact. When they start to interact, we propose that instead of shadowing being the solution to the unitarity problem, the actual growth is faster than we had estimated. If they push each other out of the way in the transverse plane, then the area will grow proportional to the number of constituents. The number of constituents is proportional to the momentum, since every parton is wee, and therefore the area  $A$  of this object will grow like the momentum,

$$A \propto \frac{P^+}{\epsilon} . \quad (18)$$

This is deeply connected to another fact – Beckenstein's observation[4] that the entropy  $S$  of any system can never be bigger than a quarter of the area,

$$S_{\text{max}} = A/4 , \quad (19)$$

in Planck units. The connection is that Beckenstein gives us a bound on the number of degrees of freedom per unit area. You cannot pack stuff more solidly per unit area than one bit of information per Planck area. And therefore, the bits which form a particle cannot stack up to more than one per unit area, and they are forced to spread out. This effectively determines the proportionality constant in eqn. (18),

$$A \sim \ell_{\text{planck}}^2 \frac{P^+}{\epsilon} . \quad (20)$$

The area law of eqn. (18) can, in fact, be proved by considering the following experiment. Take a target, a piece of foil for example, and consider what happens when you smash a billion Planck energy particle into it. When a particle get such a large momentum, much bigger than the Planck scale, we can use classical physics to describe what happens, since energy being large in gravity theory is like charge being large in QED. When the charge gets large, the electro-magnetic field becomes classical. When the momentum of a particle becomes sufficiently large, the gravitational shock wave that travels with it becomes very classical. The particle will blast out a hole from the target, and an operational measure of the size of the particle is the size of the hole. Let's go now to the center of energy frame. Let the incoming particle have energy  $E_{\text{cm}}$ . That energy stored in a system at

rest would have a Schwarzschild radius. If these two particles have an impact parameter smaller than this radius, then they will form a black hole. That is a significant collision, a process in which a large number of secondary particles come out – the Hawking radiation from the new black hole. Particle physicists would just say that a big collision happened, and the fireball heated up, and it was a very inelastic collision. Therefore, from classical physics, we know that the maximum impact parameter  $b_{\text{max}}$  at which a collision takes place is

$$b_{\text{max}} \simeq GE_{\text{cm}} = G\sqrt{S} , \quad (21)$$

where  $S$  is the Mandelstam variable. Going back into the laboratory frame,  $S = E_T P_{\text{lab}}$ , where  $P_{\text{lab}}$  is the momentum of the incoming particle. Therefore the area of the particle, effectively given by  $A = \pi b_{\text{max}}^2$ , grows like

$$A \approx \ell_{\text{planck}}^4 E_T P_{\text{lab}} . \quad (22)$$

Furthermore, by comparing (22) with (18) we find that the role of the parameter  $\epsilon$  is played by

$$\epsilon = \frac{1}{E_T \ell_{\text{planck}}^2} . \quad (23)$$

It is not surprising that the ability of a target apparatus to detect high frequencies should be limited, through the uncertainty principle, by its energy.

What does that mean for a particle falling into a black hole? This implies that it's area grows much faster than the previous string theory based argument. Instead of  $R^2$  growing like asymptotic time  $t$ , we now find

$$R^2 \sim P \sim e^{t/4MG} . \quad (24)$$

This is exactly the maximum rate of growth that a perturbation near the horizon of a black hole can spread out by causality. So we are finding this behavior by studying the longitudinal boost properties of matter, and find that it spreads over the horizon<sup>11</sup>.

## 6. Counting the Degrees of Freedom of Spacetime

There is an odd and confusing point that seems to have become clear after conversations between myself (L.S.) and Gerard 't Hooft this summer, which is that the world that we see is in a certain sense not three dimensional, but is two dimensional. In 't Hooft's words, the world is a kind of illusion, analogous to a hologram. Its information can be

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<sup>11</sup> The spreading is of course bounded by the area of the horizon. We cannot study this problem fully because once particle's transverse size gets to be of order the horizon, our flat space string theory description of its behavior breaks down. No one can follow the evolution of the string past the point where it begins to feel the curvature.

stored in two dimensions, but nevertheless can be looked at from different angles. The transformations between the description of the information as seen from different angles are called the angular conditions by light-front enthusiasts. These difficult transformations that rotate the light-front frame are those which 't Hooft thinks of as looking at a Hologram from different angles. Let us try to see more precisely what the connections are between the holographic theory and the light-front pictures that light-front community has been developing for the study of QCD.

First of all, why does one think that the world is two dimensional instead of three dimensional? The idea goes back to Beckenstein, who says that the entropy of a black hole counts the number of degrees of freedom in a region of space-time, and that this entropy is much less what you might ordinarily expect. In ordinary three dimensional (plus time) physics, we imagine space being filled with degrees of freedom. Many people would like to propose that space is either discretized or somehow regulated so that the number of degrees of freedom per unit volume is no larger than one per Planck volume  $\ell_{\text{planck}}^3$ , which is nice because then they imagine that their field theories can be finite. For the moment, let's use some kind of lattice structure like this simply to count the number of degree of freedom. If the degrees of freedom were just bits of binary information, then the number of states such a system has would be  $2^V$ , where  $V$  is the volume measured in Planck units. The number of degrees of freedom would clearly be proportional to the volume, and the maximum entropy would be

$$S = V \log 2 . \tag{25}$$

That's a plausible counting rule for all ordinary low energy physics. The basic concept, due to Beckenstein, which changes this counting rule is the following. According to Beckenstein, the entropy of a black hole is proportional to its area. And we know that most of the states in the 3-D lattice picture of space-time are high energy. We should exclude states which have so much energy inside the volume  $V$  that they form a black hole bigger than this volume. Clearly, we are over counting states if we're including states which have so much energy that the energy is bigger than a possible black hole of that size. So consider the following question – can we find in the volume  $V$  a small region of space which has an energy which is smaller than that of a black hole of volume  $V$ , but which has entropy larger than the black hole? Suppose we found this object in the center of the cubic volume. Now create a black hole around it by simply imploding a shell of matter so that this energy plus the energy in the small region makes a black hole of the appropriate size inside  $V$ . Then, we have just created a black hole around the region, and while originally the entropy of the region was bigger than the entropy of a black hole surrounding  $V$ , now that we have created the black hole, the entropy is smaller than the entropy which was in the region by assumption. This system beats the second law of thermodynamics. So if we believe the second law, as we do, then we cannot have the number of degrees of freedom in the region  $V$  which is larger than its surface area in



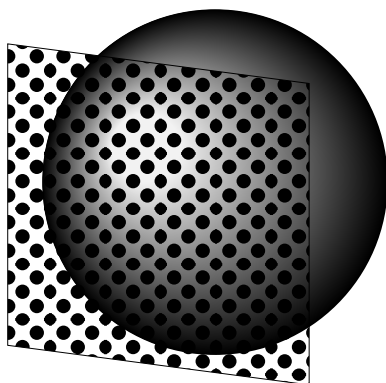


Figure 7: 't Hooft's holographic representation of the world: the pixels on the window at the edge of the sphere encode all the information on the surface of the sphere. The surface of the sphere, by an entropy argument in the text, in turn encodes all of the information inside it.

Planck units.

't Hooft says that what this means, is that in fact we have an entire description of everything inside the region if we have a sufficiently detailed map, with Planckian resolution, of everything on the surface of the volume. If we could see and watch what was going on on the surface, with sufficient detail and precision, not an infinite precision, but enough to see Planckian cells, with a binary decision in them, then we have all the information inside the region. Let's take the volume  $V$  and the sphere encompassing it to be very large, and stand near the sphere, and look at the world through the sphere. Then our view through the sphere is approximated by a plane, or window, and everything behind that window can really be coded by pixels on the window of one pixel per Plank area, with a pixel being a quantum mechanical binary bit. This is 't Hooft's two dimensional world, shown in figure 7.

## 7. Encoding Three Dimensional Information into Bits on the Screen

Let us consider a few examples of how this information might be encoded onto the screen of figure 7. The picture of the world is that it is made of tiny information 'bits', which are binary in nature, and everything in the world in made up of the same bits. Just to get started, let us first consider static configurations. We want to code the information of the static configuration on a screen. Consider the following rule. Take a light ray associated with a parton, and simply pass it through the screen perpendicularly, as in figure 8. Imagine that the screen is composed of a bunch of 'light tubes', and that the

Figure 8: A single parton projected onto the screen by light ray.

observer sees the universe by looking at the tubes. We will argue that you can never store more information along a column perpendicular to the screen than one bit per unit area. The argument basically constructs a mapping between bits on the screen and degrees of freedom in space away from the screen, using the rule of figure 9, so the final product here will be a mapping or encoding of three dimensional information on the two dimensional screen. A column density is a density per unit area where you take everything behind the screen and you simply make a long straight thin column of width  $\ell_{\text{planck}}^2$ . The amount of matter inside is the column density. There is separate information along the column, and we want to map that information to different bits on the screen without exceeding one bit of information per Planck area. Let's try to store first the maximum amount of information in the 3D region next to the screen. According to the previous section, to do this we make a black hole. The black hole has one bit per area of information on the horizon. Now let's use our rule, and take a light ray from every point on the horizon and project it onto the screen. The picture of how to do this is shown in figure 8. In thinking about the light ray lines, it is always easier to think of the light rays as coming from the screen and following geodesics down to the horizon of the black hole. The black hole forms a disk on the screen. What can we say about the mapping from the horizon to the screen? The mapping should not take an area on the horizon and project it to a smaller area on the screen. Otherwise, we would be packing more than one bit per Planck area of information onto the screen. The focusing theorem[12] guarantees that this won't happen. It says that if you follow a bundle of light rays, the second derivative of the area of the bundle with respect to the affine parameter is always negative. The more matter that is in the black hole, the more negative it is. In other words, matter tends to pull light rays together. Since these light rays start out parallel at the screen, the area of a bundle of light rays always decreases as you move away from the screen. Note that

Figure 9: A black hole projected onto the screen.

at the pole of the black hole closest to the screen, the mapping is approximately area preserving, so you know you have to pack one bit per unit area on the screen. So far, it is not surprising that we have not succeeded in filling the screen up with a higher amount of information than one pixel per Planck area. So consider putting some more stuff in behind the black hole, for example another black hole. What do the light rays do for this configuration? Gravitational lensing occurs, and light rays don't pass through black holes, they go around black holes. What you see is an Einstein ring around the disk from the closer black hole. The rays which make the ring can be traced back to the second black hole. No information is lost, even though some of the rays you might trace from the second black hole fall into the first black hole. Put another way, there is always some light ray which joins a bit on the horizon of the second black hole to the screen. There is no shadowing in gravitation. You can't hide behind a black hole. If you try to hide behind one, then someone can see your Einstein ring. The more bits you put behind the first black hole, the further out the rings will appear.

Supposing we are interested in a black hole slowly being eclipsed by a closer one. The the screen will see a sequence of patterns as shown in fig. 9. Initially, you see two disks of the two black holes, as they get closer, assuming they do not collide, the image from the one behind starts to get deformed, then at 'total eclipse' you get an Einstein ring, and finally a two disks as they draw apart. If they collide and merge, they will just form another black hole, and the density of information will still not be bigger than one bit per Planck unit on the screen.

This picture makes you think that information is behaving like a fluid which is repulsive and incompressible. Somehow gravity must have some component to it which is repulsive, always preventing you from saturating the screen with arbitrarily high information density. In this sense, bits have elbows.

Figure 10: One black hole passes behind another, producing an Einstein ring of information, similar to gravitational lensing, on the screen.

## 8. Encoding Information of Moving Particles

Now let's go a little further. We have developed a description which is necessarily a little imprecise because we want to be doing quantum mechanics, and the above was classical. The next step is to think about matter which is moving around and so *what we have to map is not points of space to the screen, but points of space-time to a 2 + 1 dimensional space-time*. We do this in the same way – take a light ray from that event, such that it hits the screen perpendicularly. So we assign to this point here a transverse position  $x_{\perp}$ . An instant on the screen corresponds to a light-front. It corresponds to everything along the light ray back into the past on a light-front. To make this more precise, we can introduce a set of light-front coordinates by gauge fixing the metric of space-time to have the form

$$ds^2 = g_{+-}dx^+dx^- + g_{+i}dx^+dx^i + g_{ij}dx^i dx^j \quad (26)$$

where the components  $x^i$  refer to transverse space and  $(x^+, x^-)$  are light-like linear combinations of the time and  $z$  coordinates. Assume that at  $x^- = \infty$  the metric has the flat form with  $g_{+-} = 1, g_{+i} = 0, g_{ij} = \delta_{ij}$ . We may identify  $x^+ = z + t, x^- = t - z$ . The screen will be identified as the surface  $x^- = \infty$ . The trajectories

$$x^+ = \text{const} , \quad x^i = \text{const} \quad (27)$$

are easily seen to be light-like geodesics and the surface  $x^+ = \text{const}$  is a light-front. We will follow the standard practice of using  $x^+$  as a time coordinate when doing light-front quantization. As we have seen it is also the time at the screen. So we are describing everything in the universe with light-front quantization, except somehow we have to suppress the  $x^-$  dependence (the distance along the light-front). This will have to be some kind of light-front quantization where  $x^-$  information is stored in a different way than we are used to in quantum field theory. What kind of ways are possible? Let us go back to a single bit stored as in figure 8. How do you distinguish the  $x^-$  position of the bit? First, recall that in quantum mechanics, we don't need to code both the position and the momentum. We will code the momentum  $p^+$  of the particle (along with its transverse position). The longitudinal momentum  $p^+$  will be encoded with discrete light-front quantization (DLCQ)[13]. Momentum comes in little individual bits<sup>12</sup> of size  $\epsilon$ , where we get to choose  $\epsilon \sim M_{\text{Planck}}$ , but one can make it even smaller<sup>13</sup>. To code the momentum, we say that a particle with a bit of minimum momentum, lights up one pixel on the screen. What about a particle with two units of momentum? It lights up two pixels. So the rule is that the information is coded by the number of bits on the screen. If there is another particle behind the first, one will expect some kind of quantum mechanical lensing effect. It is not at all obvious that we can't code everything in this way.

We can get back to regular quantum field theory when the information density is very low. Imagine going to this limit by keeping the momentum fixed in the partons but letting the pixel size on the screen get small. When the pixel size gets infinitely small, on an arbitrarily small patch of the screen, one can code any momentum one desires, and the full light-front quantization is restored. The larger the momentum of the particle, the more bits it has to have, and this picture simply tells you to think in the light-front frame of particle with momentum  $p$  as having a number of constituents proportional to  $p$ , with its area necessarily growing at least as fast as momentum. We are back where we started, in section 5 where we concluded that a particle of momentum  $p$  had to have constituents proportional to  $p$ , and that the constituents were not allowed to get into each other's way.

## 9. Discussion

What this picture is certainly telling us is that we should not be doing quantum field theory at the Planck scale, because we are over counting the degrees of freedom by a tremendous amount. And it strongly suggests that the Universe is not even a continuum  $2 + 1$  dimensional theory, because our arguments are based on a lattice picture of the storage of bits of information. So if we take this seriously, a proper description of quantum

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<sup>12</sup>Eventually we will have to take the continuum limit of this picture, but we will not discuss that problem in this lecture.

<sup>13</sup> One will have to be very careful in any real theory of this that even with this cutoff, the theory comes out to be longitudinally boost invariant.

gravity will be a discrete light-front theory (DLCQ) with a discrete transverse plane of degrees of freedom, with one bit of information per unit area of the transverse plane.

't Hooft asked the question – if one does formulate a theory like this, that anything can be mapped to a surface along any direction, what are the set of transformation rules which would allow you to look at the information from different directions? We are pointing out that this is just the light-front problem of implementing the angular conditions – how you represent the rotations of the direction of the light-front frame in a parton type description. The physics of the real three dimensional world is expected to be a representation of the three dimensional rotation group. We have to represent this group in the theory defined on the 2D screen.

Ordinary bosonic string theory has been written in a form which may help establish the relationship between string theory and the holographic principles described in this lecture [2,10]. Details can be found in the original references. These are ‘lattice string theories’, where the transverse plane is replaced by a discrete lattice with spacing  $l_s$ , and this lattice spacing is kept *fixed* throughout. A lower longitudinal momentum cutoff  $\epsilon$  is introduced so that  $p^+$  comes in discrete units.

In the short term there are a number of areas in which progress can be made. First a better understanding of the concept of fixed points and their relation to Lorentz boosts and high energy scattering is possible in ordinary field theories like QCD [14]. Secondly, for a string theory to provide an interesting candidate for a holographic theory, several missing ingredients have to be filled in. The first would be to show that the  $1/N$  expansion reproduces the bosonic string perturbation expansion. However, even if this can be done, the theory can not be analyzed nonperturbatively because of the tachyon instability. Therefore it is very important to determine if the lattice model can be supersymmetrized. Assuming this is possible, we can then attack the nonperturbative issues.

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